

# Cooling by heating in Quantum Optics Domain

D. Z. Rossatto<sup>1</sup>, A. R. de Almeida<sup>2,3</sup>, T. Werlang<sup>1</sup>, C. J. Villas-Boas<sup>1</sup>, N. G. de Almeida<sup>3</sup>

<sup>1</sup>*Departamento de Física, Universidade Federal de São Carlos,  
13565-905, São Carlos, São Paulo, Brazil*

<sup>2</sup>*UnUCET - Universidade Estadual de Goiás,  
75132-903, Anápolis (GO), Brazil and*

<sup>3</sup>*Instituto de Física, Universidade Federal de Goiás, 74.001-970, Goiânia - GO, Brazil*

## Abstract

A class of Hamiltonians that are experimentally feasible in several contexts within Quantum Optics area and which lead to the so-called cooling by heating for fermionic as well as for bosonic systems have been analyzed numerically. We have found a large range of parameters for which cooling by heating can be observed either for the fermionic system alone or for the combined fermionic and bosonic systems. Finally, analyzing the experimental requirements we conclude that cooling by heating is achievable with nowadays technology, especially in the context of trapped ions/cavity QED, thus contributing to understand this interesting and counter intuitive effect.

Recently, two schemes were proposed for cooling by heating [1, 2], *i.e.*, a given physical system in contact with a thermal reservoir has decreased its energy when one increases the temperature of the reservoir. A. Mari *et al.* [1] introduced the idea of cooling a quantum system using incoherent thermal light. They proposed a scheme based on an optomechanical system, demonstrating that by driving the system with a thermal noise the interaction with other modes can be enhanced to assist in cooling the optomechanical system. In another work, B. Cleuren, B. Rutten, and C. Van den Broeck [2] proposed a scheme to cool a system powered by photons. Their systems are based on a nanosized solid state device, with no moving parts and no net electric currents, which can be refrigerated directly by using thermal photons.

In this brief report we investigate numerically a class of well known Hamiltonians in the quantum optics domain and show that these Hamiltonians can lead to cooling by heating. Differently from both schemes above, which investigate the cooling by heating in solid state or optomechanical devices, our work brings this striking effect to the quantum optics context where techniques to manipulate systems at the individual atomic and bosonic scale is daily presented, thus opening the possibility to experimentally observe this phenomenon in a very controllable scenario.

*Model.* In order to find out a system which allows us to cool it by raising the temperature of its reservoir, firstly we must note that all the systems which thermalize with the environment can not present this phenomenon (for example a single two-level atom or a single bosonic mode interacting with a thermal reservoir). Thus, to see cooling by heating some external force must be employed to drive the system out of equilibrium with the environment. To this end we explore the well known generalized anti-Jaynes-Cummings model (JCM) (which will be derived bellow), in the coupling regime where the effective Rabi frequency (atom-boson coupling) is much smaller than the bosonic and atomic transition frequencies. To implement such Hamiltonians in the trapped ions domain, for instance, one can use a two-level ion characterized by the transition frequency  $\omega_0$  between the ground  $|g\rangle$  and excited  $|e\rangle$  states and trap frequency  $\nu$  (bosonic mode) [3]. The transition  $|g\rangle \leftrightarrow |e\rangle$  is driven by a classical field of frequency  $\omega_L$ , wave vector  $k_L = \omega_L/c$ , and Rabi frequency  $\Omega$  [3]. In the Schrödinger picture, the Hamiltonian which describes such a system reads ( $\hbar = 1$ )

$H = H_f + H_a + H_{int}(t)$ , with  $H_f = \nu a^\dagger a$ ,  $H_a = \omega_0 \sigma_z/2$  and

$$H_{int}(t) = \frac{\Omega}{2} \sigma_- e^{i(k_L \hat{x} + \omega_L t)} + H.c., \quad (1)$$

$\sigma_+(\sigma_-)$  being the usual raising (lowering) Pauli operator for a two-level atomic system,  $\sigma_z = \sigma_+ \sigma_- - \sigma_- \sigma_+$ ,  $a$  ( $a^\dagger$ ) is the annihilation (creation) operator in the Fock space for the bosonic mode (vibrational motion of the ion),  $H.c.$  means Hermitian conjugate,  $k_L \hat{x} = \eta_L (a + a^\dagger)$ , with  $\eta_L = k_L / \sqrt{2m\nu}$  being the Lamb-Dicke parameter [3]. Working in the limit  $\eta_L \ll 1$  and applying the rotating wave approximation, the Hamiltonian  $H$  in the interaction picture can be written as [3]

$$H_I = g_k (\sigma_- a^k + \sigma_+ a^{\dagger k}), \quad k = 0, 1, 2. \quad (2)$$

By adjusting the frequency  $\omega_L$  on resonance with the two-level ion we can have the carrier interaction ( $k = 0$ ,  $g_0 = \Omega/2$ ); adjusting  $\delta = \omega_L - \omega_0 = k\nu$ , we can also have the first ( $k = 1$ ,  $g_1 = i\Omega\eta_L/2$ ) and second ( $k = 2$ ,  $g_2 = -\Omega\eta_L^2/4$ ) blue sideband interactions [3]. The quantum of vibrational energy of the center of mass of the ion is then described by  $a^\dagger a$ . In quantum optics area, the dynamics of this model under Born and Markov approximations (weak system-reservoir coupling) is provided by the master equation formalism [4], which for the Hamiltonian (2) reads

$$\begin{aligned} \frac{\partial \rho}{\partial t} = & -i [H_I, \rho] + \kappa (n_{th} + 1) \mathcal{D}[a] \rho + \kappa n_{th} \mathcal{D}[a^\dagger] \rho \\ & + \gamma (m_{th} + 1) \mathcal{D}[\sigma_-] \rho + \gamma m_{th} \mathcal{D}[\sigma_+] \rho \end{aligned} \quad (3)$$

where  $\kappa$  and  $\gamma$  are the spontaneous emission rates for the vibrational motion and internal levels of the ion, respectively,  $n_{th}$  ( $m_{th}$ ) is the mean number of phonons (photons) of the reservoir coupled to the vibrational mode (internal levels of the ion), and  $\mathcal{D}[A] \rho \equiv 2A\rho A^\dagger - A^\dagger A \rho - \rho A^\dagger A$  [5].

Below we proceed to solve numerically (or analytically, for the carrier interaction) this master equation to obtain the steady state of the system (at  $t \rightarrow \infty$  or  $\partial \rho / \partial t = 0$ ), in order to be able to calculate the corresponding thermodynamical properties. To this aim, we firstly have to note that the master equation can give rise to an infinity set of coupled differential equations for the elements of the density matrix of the whole system. Then, to solve it numerically, first we must truncate the Fock basis of the bosonic field somewhere. This truncation depends on the mean number of excitations in the vibrational mode, *i.e.*,

the matrix elements corresponding to highly excited Fock states (compared to the mean number of excitation of the vibrational mode) must be virtually zero. We then integrate numerically the system of coupled differential equations for the elements of the density matrix of the system following the method presented in [6]. As we are working with two distinct reservoirs, there will be different response functions: one for the atom and another for the vibrational mode [7]. Working with the Hamiltonian (2) we can distinguish three situations where cooling by heating (i.e., by raising the temperature of the reservoir) can be observed:

- (i) looking at the variation of the internal energy of the ion only;
- (ii) looking at the variation of the vibrational energy of the ion;
- (iii) looking at the variation of both internal and vibrational energies of the ion.

*Variation of the internal energy of the ion only.* Firstly we assume a carrier interaction ( $k = 0$ ) in Eq.(2) which corresponds to a single two-level ion driven by a classical field. Since the dynamics of the system does not involve the vibrational mode, we can fix  $\kappa = 0$  without loss of generality. From Eq.(3), we can easily obtain the average internal energy  $E_a = \langle H \rangle = \langle H_a + H_{INT} \rangle = \langle H_a \rangle$  of the ion in the steady state as a function of the mean number of thermal photons (temperature of the reservoir). Then we can calculate the response function ( $C_a$ ) of the internal energy with respect to the temperature of its reservoir ( $T$ ) which we define as

$$C_a = \frac{\partial E_a}{\partial T}. \quad (4)$$

This equation resembles the usual definition of specific heat. However, note that the temperature appearing in the equation above is the one of the reservoir, which is different of the effective temperature of the system since it is not in thermal equilibrium with its environment. With the steady state solution for the internal energy  $E_a$ , we can analytically derive the response function

$$C_a = -2k_B m_{th} (m_{th} + 1) \times \left[ \ln \left( \frac{m_{th} + 1}{m_{th}} \right) \right]^2 \frac{[2(g_0/\gamma)^2 - (2m_{th} + 1)^2]}{[2(g_0/\gamma)^2 + (2m_{th} + 1)^2]^2}, \quad (5)$$

$k_B$  being the Boltzmann constant. Clearly, we see from Eq.(5) that  $C_a \leq 0$  for

$$m_{th} \leq \frac{1}{\sqrt{2}} \frac{g_0}{\gamma} - \frac{1}{2}. \quad (6)$$

Note that  $C_a \rightarrow 0$  when  $m_{th} \rightarrow 0$  (similar to what occurs for the third law of thermodynamics) or  $m_{th} \rightarrow \infty$  (system saturation). For a sample of  $N$  non-interacting atoms [8], the response function is  $C_N = NC_a$  and then, the negative response can be observed even for an ensemble of two-level atoms.

*Variation of the vibrational and internal energy of the ion.* Considering the ion coupled to the vibrational mode, we note that using the average energy of an individual system instead of the total energy (sum of the subsystems and interaction average energies) does not change our conclusions, since there is a region where the response function, Eq.(4), becomes negative for both systems simultaneously and  $Tr(H_I\rho) = 0$  in all cases studied here,  $H_I$  being the interaction Hamiltonian (2). To see that this is so, in Fig. 1(a) ( $k = 1$ ) and Fig. 2(a) ( $k = 2$ ) we plot the stationary average energy for both the bosonic mode ( $\langle H_f \rangle / \nu$ ) and the atomic system ( $\langle H_a \rangle / \omega_0$ ). In all simulations we assumed zero atomic energy in the lower state  $|g\rangle$ . To reliably calculate the region where cooling by heating can occur, we have limited our numerical analysis to the range  $0 \leq g_k, \kappa \leq 2\gamma$  ( $k = 1, 2$ ). Also, in all figures, the average atomic energy was multiplied by a factor of ten for the sake of clarity. Assuming the two reservoirs at a common temperature ( $m_{th} = n_{th}$ ), first we set  $\kappa = 0.1\gamma$  and  $g_1 = 1.0\gamma$  in Fig. 1(a), and  $\kappa = 0.1\gamma$  and  $g_2 = 0.2\gamma$  in Fig. 2(a). Remarkably, note from those figures that there is a region where the response to the rising reservoir temperature of both atomic and bosonic system is negative (falling energy), thus supporting our assertion that cooling by heating can be observed even if we adopt a definition, different from Eq.(4), taking into account the total energy. Also, note that the final temperature of the bosonic system differs from that of its reservoir ( $\langle a^\dagger a \rangle \neq n_{th}$ ), thus indicating the existence of non-equilibrium steady states [7, 9]. It is important to mention that it is not surprisingly to have a *non-equilibrium* steady state once the system is driven by an external force (the external laser).

In Fig. 1(b) (Fig. 2(b)) we plot the response function versus  $n_{th}$  to the bosonic and the atomic systems for the model  $k = 1$  ( $k = 2$ ). Both figures show that cooling by heating for the bosonic system can occur in a wider region than that for the atomic system.

Let us now explore the fact that the reservoirs for the atomic and bosonic systems can have different mean number of thermal photons. This is particularly relevant for trapped ion experiments since the transition frequency  $\omega_0$  (of the order of few GHz) of the electronic levels involved are usually much bigger than the frequency  $\nu$  of the ionic motion ( $\sim$ MHz)

[10], resulting in different mean number of thermal photons for the electronic levels and ionic motion for a given temperature. In Fig. 3(a) we show the behavior of the response function for the atomic system when the mean photon number of the bosonic reservoir is fixed at  $n_{th} = 0.0, 1.0, 2.0$ , for the model  $k = 1$ , using the same parameters as those in Fig. 1. Note that cooling by heating can occur when  $m_{th} \lesssim 1$  irrespective of the fixed  $n_{th}$ . Fig. 3(b) does the same for the model  $k = 2$ , with the parameters used in Fig. 2. Cooling by heating can now occur when  $m_{th} \lesssim 0.5$ .

It is noteworthy that when we fix the average number of thermal photons for the atomic reservoir and investigate the behavior of the response function to the bosonic system as a function of temperature, we do not see regions where cooling by heating can occur. Besides, for the range of parameters used in our numerical simulations, we have found regions where cooling by heating can occur in the atomic system for some values of the effective Rabi frequency  $g_k$ , irrespective of the ratio  $\kappa/\gamma$ .

On the other hand, to the system under study, to observe cooling by heating for the bosonic system for some effective Rabi frequency  $g_k$ , not only the reservoirs must have the same average photon number ( $m_{th} = n_{th}$ ) but the atomic decay must be stronger than the bosonic mode decay, which our numerical simulations point to the ratio  $\kappa/\gamma \lesssim 0.3$  for the model  $k = 1$  and  $\kappa/\gamma \lesssim 0.4$  for the model  $k = 2$ . In a first moment, one could think that we should have the same response regardless the bosonic or the fermionic system, once the equation of motion (3) is completely symmetric on the fermionic and on the bosonic operators. However, the nature of those operators are completely different, i.e., the fermionic operators are restricted to a two-dimension Hilbert space while the bosonic ones are in a infinite Hilbert space. So, the physical difference is the number of accessible states of each subsystem: the fermionic subsystem has only two accessible states and the bosonic subsystem can access infinite states.

*Experimental proposal:* We now comment on the parameters appearing in the effective Hamiltonians discussed above and how cooling by heating could be observed with the nowadays technology. In the trapped ions domain, for instance, Hamiltonians Eq.(2) were obtained and used to engineer nonclassical motional states [3]. For the anti-JCM ( $k = 1$ ) and the so-called two-phonon anti-JCM ( $k = 2$ ), the effective couplings are, respectively,  $|g_1| = |\eta_L \Omega|/2$  and  $|g_2| = |\eta_L^2 \Omega|/4$ , where  $\eta_L$  is the Lamb-Dicke parameter and  $\Omega$  is the Rabi frequency of the classical field driving the two-level ion, which can easily be adjusted

[3]. For the hyperfine ground states of a single  ${}^9\text{Be}^+$  ion, one can adjust  $\eta_L = 0.2$ , thus lying in the Lamb-Dicke regime [10]. We note that typical starting values of the average number of thermal phonons in the mode of interest are between 0 and 2 and the decay rate of the vibrational motion of the ion can be much smaller than  $g_k$  [11]. Thus, a trapped ion seems to be an appropriate physical system to observe cooling by heating. The existence of cooling by heating in this context provides an interesting and counter-intuitive application: the ion motion can be reduced as the reservoir temperature increases. The anti-JCM can also be engineered in a cavity QED setup [12]. As the usual atom-field coupling in the microwave domain is  $\lambda \sim 10^5 \text{s}^{-1}$ , the effective coupling for the anti-JCM can be  $g_1 \sim 10^3 \text{s}^{-1}$  [12]. The cavity decay rate  $\kappa$  ranges from  $10 \text{s}^{-1}$  to  $10^2 \text{s}^{-1}$  [13] and, therefore, we easily attain the condition  $0 \leq g_k/\kappa \lesssim 10$ . Taking into account realistic temperatures, the effective mean occupation number at the microwave frequency has to be  $n_{th} \sim 0.7$ , according to QED cavity experiments [14]. This mean number of thermal photons can be reduced down to 0.1 by sending atoms resonantly with the cavity mode to absorb the thermal field [14].

*Conclusion.* We studied a class of Hamiltonians well-known in the quantum optics domain and showed that cooling by heating can occur for a large range of parameters, including some achievable by present day techniques. We numerically solve the master equation and calculate the response function of the internal energy for systems interacting with a thermal bath when varying their corresponding reservoir temperature: *a*) a single two-level atom (or even a sample of  $N$  two-level atoms) driven by a classical field and *b*) a bosonic mode interacting with a two-level ion/atom. We hope this work will trigger a search for experimental verification of cooling by heating in the quantum optics area, thus strongly contributing to understand this interesting and counter intuitive effect.

## Acknowledgments

The authors acknowledge the financial support from the Brazilian agencies CNPq and Brazilian National Institute of Science and Technology for Quantum Information (INCT-IQ). C. J. V. B. also acknowledges support from FAPESP (Proc. 2012/00176-9).

---

[1] A. Mari and J. Eisert, Phys. Rev. Lett. **108**, 120602 (2012).

- [2] B. Cleuren, B. Rutten, and C. Van den Broeck, Phys. Rev. Lett. **108**, 120603 (2012).
- [3] D. Leibfried *et al.*, Rev. Mod. Phys. **75**, 281 (2003).
- [4] H.-P. Breuer and F. Petruccione, *The theory of Open Quantum Systems* (Oxford University Press, Oxford, 2007).
- [5] G. Lindblad, Commun. Math. Phys. **48**, 119 (1976)
- [6] S. M. Tan, J. Opt. B: Quantum Semiclass. Opt. **1**, 424 (1999).
- [7] R. K. P. Zia, E. L. Praestgaard, and O. G. Mouritsen, American Journal of Physics **70**, 384 (2002).
- [8] The Hamiltonian for  $N$  identical non-interacting atoms is obtained by replacing  $\sigma_+ \rightarrow S_+ = \sum_{i=1}^N \sigma_+^i$  in Eq. (2) for  $k = 0$  ( $S_- = S_+^\dagger$ ). Here  $\sigma_+^i$  is the Pauli operator acting on the  $i$ -th atom. The calculation of the response function for  $N$  atoms is almost identical to the calculation performed for a single atom
- [9] D. Lynden-Bell, Physica A **263**, 293 (1999).
- [10] D. M. Meekhof *et al.*, Phys. Rev. Lett. **76**, 1796 (1996).
- [11] Q. A. Turchette *et al.*, Phys. Rev. A **61**, 063418 (2000).
- [12] M. França Santos, E. Solano, and R. L. de Matos Filho, Phys. Rev. Lett. **87**, 093601 (2001).
- [13] S. Gleyzes *et al.*, Nature **446**, 297 (2007).
- [14] J. M. Raimond, M. Brune, and S. Haroche, Rev. Mod. Phys. **73**, 565 (2001)..

### Figure Caption

Fig. 1: (color online) (a) Average energy for atomic ( $\langle H_a \rangle / \omega_0$  - dashed line (x10)) and bosonic systems ( $\langle H_f \rangle / \nu$  - solid line) versus a common mean number of thermal photons  $n_{th} = m_{th}$ , for the model  $k = 1$ . (b) Response function versus mean number of thermal photon. The cooling by heating can occur for  $m_{th} = n_{th} \lesssim 1.4$  to the bosonic and  $m_{th} = n_{th} \lesssim 0.9$  to the atomic system. The parameters used are  $\kappa = 0.1\gamma$  and  $g_1 = 1.0\gamma$ .

Fig. 2: (color online) (a) Average energy for atomic (dashed line (x10)) and bosonic systems (solid line) versus a common mean number of thermal photons  $n_{th} = m_{th}$ , for the model  $k = 2$ . The mean value of the interaction Hamiltonian  $H_I$  (not shown in this figure) is always zero. (b) Response function versus mean number of thermal photons. The cooling by heating can occur for  $m_{th} = n_{th} \lesssim 1.2$  to the bosonic and  $m_{th} = n_{th} \lesssim 0.4$  to the atomic system. The parameters used are  $\kappa = 0.1\gamma$  and  $g_2 = 0.2\gamma$ .



Fig. 3: (color online) Response function to the atomic system versus the average photon number  $m_{th}$  of its reservoir when fixing the average photon number of the bosonic reservoir:  $n_{th} = 0.0$  (solid line),  $n_{th} = 1.0$  (dashed line) and  $n_{th} = 2.0$  (dotted line), for the model (a)  $k = 1$ , using  $\kappa = 0.1\gamma$  and  $g_1 = 1.0\gamma$ ; and (b)  $k = 2$ , using  $\kappa = 0.1\gamma$  and  $g_2 = 0.2\gamma$ .

FIGURE 1

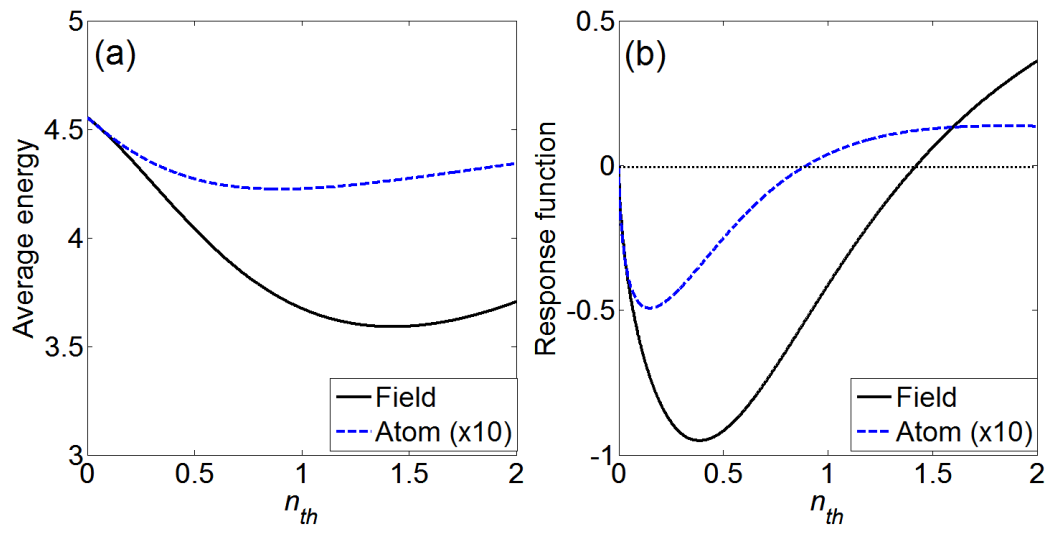


FIGURE 2

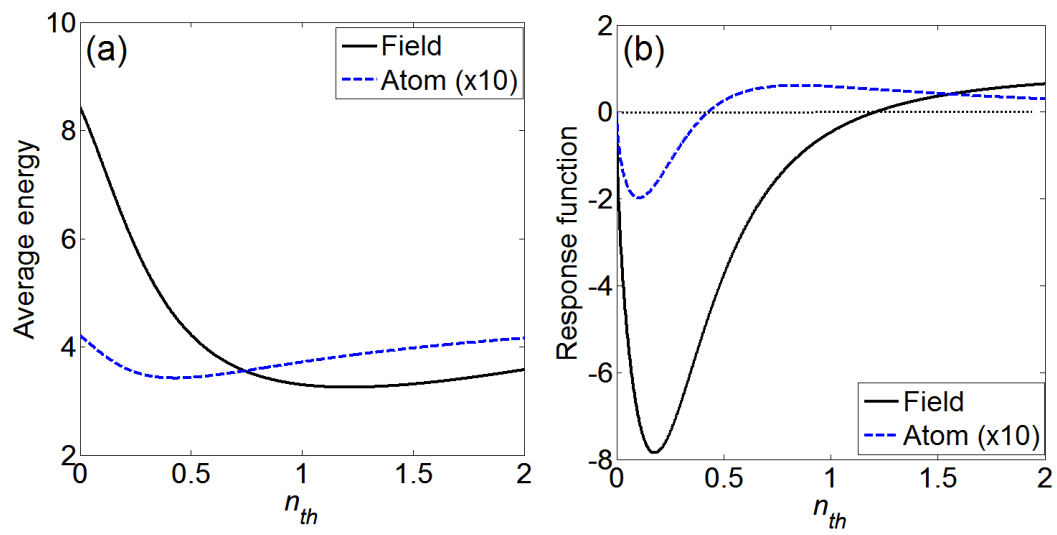


FIGURE 3

